Let $\vec{b} = 3\vec{i} - 6\vec{j} - 2\vec{k}$ and $\vec{c} = -2\vec{j} + \vec{k}$. SCORE: _____ / 55 PTS

Let L be the point $(5, -3, 4)$.

[a] Write $3\vec{c} - 2\vec{b}$ in component form.

$$3\langle 0,-2,1\rangle - 2\langle 3,-6,-2\rangle = \langle 0,-6,3\rangle - \langle 6,-12,-4\rangle$$
$$= \langle -6, 6, 7\rangle$$

[b] Find a vector of magnitude 3 in the same direction as $\vec{b}$.

$$3\left(\frac{1}{\|\vec{b}\|}\right)\vec{b} = 3\left(\frac{1}{\sqrt{49}}\right)\langle 3,-6,-2\rangle = \frac{3}{7}\langle 3,-6,-2\rangle$$
$$= \left\langle \frac{9}{7}, \frac{-18}{7}, \frac{-6}{7}\right\rangle$$

[c] Find a unit vector perpendicular to both $\vec{b}$ and $\vec{c}$.

$$\vec{b}\times\vec{c} = \langle 3,-6,-2\rangle \times \langle 0,-2,1\rangle$$
$$= \langle -6-4, 0-3, -6-0\rangle$$
$$= \langle -10,-3,-6\rangle$$
$$\frac{1}{\|\vec{b}\times\vec{c}\|}(\vec{b}\times\vec{c}) = \frac{1}{\sqrt{145}}\langle -10,-3,-6\rangle = \left\langle \frac{-10}{\sqrt{145}}, \frac{-3}{\sqrt{145}}, \frac{-6}{\sqrt{145}}\right\rangle$$

[d] Find the equation of the plane parallel to both $\vec{b}$ and $\vec{c}$, and passing through point L.
Write your final answer in general form $Ax + By + Cz + D = 0$.

$$-10(x-5) - 3(y+3) - 6(z-4) = 0$$
OR $$10(x-5) + 3(y+3) + 6(z-4) = 0$$
$$10x + 3y + 6z - 65 = 0$$

[e] Let ℓ_1 be the line with parametric equation $x = 3t - 5, \quad y = 6 - 2t, \quad z = t + 8$.
Find the symmetric equation of the line parallel to ℓ_1 and passing through L.

$$\frac{x-5}{3} = \frac{y+3}{-2} = \frac{z-4}{1}$$

OR $$\frac{x-5}{3} = -\frac{y+3}{2} = z-4$$

If $< 2, a, -12 >$ is parallel to $< b, -8, 9 >$, find the values of a and b. **SCORE: _____ / 10 PTS**

$\langle 2, a, -12 \rangle = k \langle b, -8, 9 \rangle$

$2 = kb$

$a = -8k$

$-12 = 9k \longrightarrow k = -\frac{4}{3}$

$a = -8\left(-\frac{4}{3}\right) = \frac{32}{3}$

$2 = -\frac{4}{3}b \longrightarrow b = -\frac{3}{2}$

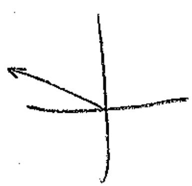
Let $\vec{r}$ be the vector with magnitude 4 and direction angle $\frac{\pi}{6}$. Let $\vec{s} = < -4\sqrt{3}, 4 >$. **SCORE: _____ / 20 PTS**

[a] Find the component form of $\vec{r}$.

$\langle 4\cos\frac{\pi}{6}, 4\sin\frac{\pi}{6} \rangle = \langle 2\sqrt{3}, 2 \rangle$

[b] Find the direction angle of $\vec{s}$.

$\tan^{-1}\frac{4}{-4\sqrt{3}} = \tan^{-1} -\frac{\sqrt{3}}{3} = -\frac{\pi}{6}$ IN Q_4



$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

[c] If $\vec{r}$ represents a force, and $\vec{s}$ is the movement of an object that the force is applied to, find the work done.

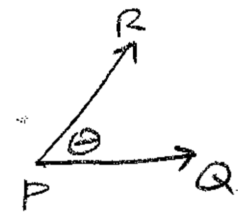
$\langle 2\sqrt{3}, 2 \rangle \cdot \langle -4\sqrt{3}, 4 \rangle = -8(3) + 8 = -16$

Let P be the point $(-2, 3, -5)$. Let Q be the point $(-8, -1, -3)$. **SCORE: _____ / 65 PTS**

Let R be the point such that $\overrightarrow{PR} = <1, 3, 2>$.

[a] Find the co-ordinates of R.

$$(-2+1, 3+3, -5+2) = (-1, 6, -3)$$



[b] In the triangle ΔPQR, find the measure of angle $\angle RPQ$.

$$\overrightarrow{PQ} = \langle -8 - -2, -1-3, -3 - -5 \rangle = \langle -6, -4, 2 \rangle$$

$$\cos^{-1} \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \cos^{-1} \frac{-6-12+4}{\sqrt{56}\sqrt{14}} = \cos^{-1} \frac{-14}{2(14)} = \cos^{-1} -\frac{1}{2}$$

$$= \frac{2\pi}{3}$$

[c] Write $\overrightarrow{PR}$ as the sum of two orthogonal vectors, one of which is the projection of $\overrightarrow{PR}$ onto $\overrightarrow{PQ}$.

$$\frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\overrightarrow{PQ} \cdot \overrightarrow{PQ}} \overrightarrow{PQ} = \frac{-14}{56} \langle -6, -4, 2 \rangle = -\frac{1}{4} \langle -6, -4, 2 \rangle = \langle \tfrac{3}{2}, 1, -\tfrac{1}{2} \rangle$$

$$\langle 1, 3, 2 \rangle = \langle \tfrac{3}{2}, 1, -\tfrac{1}{2} \rangle + \langle -\tfrac{1}{2}, 2, \tfrac{5}{2} \rangle$$

[d] Find the area of the triangle ΔPQR.

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle -6, -4, 2 \rangle \times \langle 1, 3, 2 \rangle$$

$$= \langle -8-6, -(-12-2), -18+4 \rangle$$

$$= \langle -14, 14, -14 \rangle$$

$$\frac{1}{2} \| \overrightarrow{PQ} \times \overrightarrow{PR} \| = \frac{1}{2}(14) \| \langle -1, 1, -1 \rangle \| = 7\sqrt{3}$$

[e] Find the equation of the plane passing through P, Q and R. **Write your final answer in general form** $Ax + By + Cz + D = 0$.

$$-14(x+2) + 14(y-3) - 14(z+5) = 0$$

OR $(x+2) - (y-3) + (z+5) = 0$

$$x - y + z + 10 = 0$$